Xtreme Arithmetic

Education in Survival Mode

By

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ARTHRITIC ARITHMETIC

I sent this Internet joke to my friend, John Harrison, inventor of the Numdrum. He wrote back, “I don’t know whether to laugh or cry.” What do you think?

> Last week I purchased a burger for $1.58. I handed the cashier $2.00 and started digging for some change. I pulled out 8 cents and gave it to her. She stood there with $2 and 8 cents. She looked bewildered, holding the nickel and 3 pennies, while looking at the screen on her register.
>
> I sensed her discomfort and tried to tell her to just give me two quarters, but she hailed the manager for help. While he tried to explain the transaction to her, she burst into tears.
>
> The incident got me thinking about how our kids were learning math in school (or not).
>
> Teaching Math In 1950: A logger sells a truckload of lumber for $100. His cost of production is 4/5ths of the price. What is his profit?
>
> Teaching Math In 1960: A logger sells a truckload of lumber for $100. His cost of production is 4/5ths of the price, or $80. What is his profit?
>
> Teaching Math In 1970: A logger exchanges a set "L" of lumber for a set "M" of money. The cardinality of set "M" is 100. Each element is worth one dollar. Make 100 dots representing the elements of the set "M." The set "C," the cost of production, contains 20 fewer points than set "M." Represent the set "C" as a subset of set "M." Answer this question: What is the cardinality of the set "P" of profits?
>
> Teaching Math In 1980: A logger sells a truckload of lumber for $100. His cost of production is $80 and his profit is $20. Your assignment: Underline the number 20.
>
> Teaching Math In 1990: By cutting down beautiful forest trees, the logger makes $20. What do you think of this way of making a living? Topic for class participation after answering the question: How did the forest birds and squirrels feel as the logger cut down the trees? (There are no wrong answers)
>
> Teaching Math In 2000: A logger sells a truckload of lumber for $100. His cost of production is $120. How does Arthur Anderson determine that his profit margin is $60?
>
> Teaching Math in 2005: El hachero vende un camion carga por $100. La cuesta de production es....

Whether that joke is funny or not depends on how emotionally involved you are with the deplorable state of education going on in our school systems. If you are one of those people who have trouble spelling words or balancing your checkbook, then you probably think it is normal for a high school or college graduate to struggle with such problems..

We are all thankful, of course, that our numbering system came from Arabia instead of Rome. How would you like to find the sum of XIII and MCIXIV?

Then why can’t our children count change, balance a checkbook, keep track of their credit card debt, or understand how a mortgage works? They did not learn the best way to use the numbers we got from Arabia. It is a fact that other countries—notably India, Switzerland and Germany—produce mathematicians we deem brilliant. Could it be that we were just taught a different system? When we fall behind our European friends in education, what is our thought pattern? Rather than saying, “Let’s find out what they are doing differently and do the same thing,” we tend more often to assume such people are brilliant. We simply cannot accept the premise that maybe—just maybe—we made the process too difficult.

Please permit me one absurd example that still makes me laugh. Several years ago the United States decided we should base our measurements on the decimal system, as they do in Europe. We immediately set out to teach everybody how to convert inches and yards into centimeters and meters, as if that were necessary. People rebelled, as they should, at such nonsense. All we needed to do was make new containers, new mileage indicators. No problem. We did not need to know how to convert; we only needed to convert and move forward. Instead, we have buried this issue for decades under a cloak of—dare I say it?—ignorance.

As a computer programmer a few years ago, I was introduced to the first of the Indian experts who arrived here to do software development. In my estimation, of course, they were brilliant. (Yes, I was guilty of operating under the same false paradigm, that I was somehow inferior.) Some of you may remember that the space programs of both the United States and the USSR were successful only through the efforts of German scientists who escaped their Nazi tethers after WW II. The Theory of Relativity was put forth by a German Scientist. And if you remember that era, you also remember a time when we thought all the genius rocket scientists came from Germany.

If your child is failing in math and you have the money, you can send him or her to the Mathematical Institute in Zurich, Switzerland. Upon returning from there, your child will be able to perform feats of wizardry you cannot imagine. The school was founded by the late Jakow Trachtenberg, a Russian. When communists came to Russia, is 1917, young Trachtenberg fled to Germany. In 1934, he was forced to flee from Germany to Yugoslavia. Hitler’s Gestapo agents caught up with him there and sent him to one of the most ruthless concentration camps. To survive, Trachtenberg retreated into his private world of mathematics, where he developed a way to do basic math without paper. Finally successful in an escape, he fled to Switzerland. There he perfected his system, taught it to children, and started his school.

“As these youngsters became proficient in handling numbers, *they achieved a poise and assurance that transformed their personalities* and they began to spurt ahead in all their studies. The feeling of accomplishment leads to greater effort and success.”* (Italics mine.)

This observation is a common theme among the subjects Xtreme Ed recommends. It is vital for a child to accomplish something quickly. It is vital for a child to gain a sense of self-worth by being successful at something early.

* The Trachtenberg Speed System of Basic Mathematics, translated and adapted by Ann Cutler and Rudolph McShane. 1960. Doubleday &Company, Inc., Garden City, New York

There is a relatively new system of mathematics called Vedic Math that claims to be a recent discovery of ancient Vedic teachings. That claim may be true, but I have found Vedic math to be remarkably similar in function to the Trachtenberg system. Trachtenberg's schools were operating for years before Vedic math was "discovered." Before Trachtenberg started his school, before 1923, when my father left school to help support his family, he had been taught "head math" in school. Regarding basic math education, then, I tend to agree with the adage, "There is nothing new under the sun."

Vedic math is similar to the Trachtenberg system, but it is presented in a different way. Other recent arrivals to the "math education" game include the "lattice" system and the Kumon system from Korea, both of which are being tried in the United States. Not so many years ago parents were dismayed when their children learned "base 8" or "base 16" math. My own approach surely bears similarities to Trachtenberg and Vedic math, because some similarity cannot be avoided. Like the "head math" my father learned, however, my presentation is easier to understand than the others, and less confusing.

The systems I have mentioned all have one thing in common. They all try to explain how their system works, using complex algebraic formulae to demonstrate. That is fine if you are a math major. You can look up the information on the Internet, or buy the book on Trachtenberg system if you want to understand math to that depth. If you just want to teach a second grader how to do long addition, however, you don't need that explanation any more than we need to convert pounds to liters. Like transferring to the metric system, you need to just do it. You need this book.

The approach I have taken is very simple. For each section—addition, subtraction, multiplication and division—the instructions are easy to understand.

- Here is what you need to know before you start this section.
- Here is the way it works.

Speed math, or head math, is easy to learn and simple to do. With practice, you will gain speed. With practice, you will reduce workspace. With practice, you may even revive head math as the popular pastime it was nearly 100 years ago.

There is a tactile early math tool that I recommend. Designed by John Harrison, it is called Numdrum and is an ingenious way to introduce young children to numbers, including addition, subtraction and multiplication tables. You may learn more about it at www.numdrum.com. The following section will provide a brief introduction to that tool. If your reaction is "Why didn't I think of that?" you are not alone. In my opinion, the simplicity and power of Numdrum is the stuff of genius.

USE NUMDRUM,
PRE-SCHOOL THROUGH GRADE ONE

The Numdrum was originally conceived as a simple aid to adding and subtracting double figure numbers for a particular 8-year-old who was having trouble. Since that time it has quickly evolved into a powerful visual aid for even pre-school children.

Numdrum consists of a number line from zero to 139, wound round a tube. There are ten numbers per turn of the tube. The resulting appearance is similar to a number square, in that vertical movement is in jumps of 10, horizontal movement in units.

There is a version of the Numdrum called a Numring that is more appropriate for pre-school and kindergarten. It is a larger tube, with larger numbers from 0 to 30. Early concepts about numbers can be taught using Numring—finding a number, or counting up and down, some concepts of adding and subtracting. When a student begins with Numring and graduates to Numdrum in first grade, half the job is done!

The important difference between the Numdrum and other visual aids used in the early grades illustrate the value of Numdrum.

In the beginning there is the number line. It seems generally agreed that this simple model is a good starting point to introduce the number system, showing its continuity and indefinite extendibility. It lacks structure, however, and gets unwieldy as numbers grow larger. Numbers grow larger as they progress from left to right.

The next step in the presentation of numbers is usually the number square, with three obvious flaws.

1. Numbers progress downward with increasing value, contrary to accepted mathematical convention.

2. Numbers usually finish at 99, preventing children from observing the transition from two the three digits. For a young child, moving from two to three digits is a trauma trigger.

3. The numbers are arranged in individual lines. There is no obvious place to go at the end of a line. Young children must find the correct line to go to next, and this is often a cause for errors in judgment at this age.

Created in 1999, and already quite popular in Great Britain, the Numdrum provides children with visual reinforcement of all the correct concepts and none of the flaws.

1. The numbers progress in an upward direction with increasing size. This conforms to established mathematical convention, and avoids the confusion that children face

when they come to draw graphs and bar charts. On the Numdrum, each succeeding number is truly higher than its predecessor, for any child to see.

2. The numbers continue well above 100, avoiding the impression given by the number square that there is some invisible barrier at 99 to prevent numbers higher than 100 being displayed. Thus a major hurdle faced by all children is avoided when they go from two-digit to three-digit numbers.

3. The Numdrum has no edges to form an obstacle when counting between 9 and 10. Unlike the number square, there is no boundary between 9, 10 and 11. Numbers progress smoothly upward from 0 to 139. There is no line to find.

Together, these three concepts build a powerful understanding of how numbers work. The net result is that children learn the proper concepts faster, with no conflicting mental baggage to hold them back.

But there is more, much more, to Numdrum.

Find a Number

Finding a number is the first challenge for children. Suggest a number and have the children raise their hands when they have found it. After a few tries, a few hands will go up quicker than others. Ask, “Who knows a good way to find numbers quickly?”

One of those whose hands were raised early will usually describe the concept you want them to learn. “Look for the number ending on the bottom row of the Numdrum and then move up the column until you find the number.”

Soon everybody in the class will be able to locate numbers quickly. They have learned how to count by tens, without you, the teacher, telling them how to do it. It’s just a game to them, a discovery process, but they understand the concept.

Addition

Try simple addition problems, adding single digits.

Choose the number 6. Have the children add three to that number by starting at the 6 and counting to the right one, two, three. They should all be pointing to the 9.

This much is similar to the number line. But it stops there. Now you start with 26. Add three to 26 by counting to the right one, two, three. The sum is 29. Your students have just learned that adding single digits to a number is no more difficult when there is one digit or two digits.

How about finding 126? Add three to get 129. Can they still do that? Of course. To add three, count three to the right.

The second hurdle is to pass the 9. Let's see if they can start with 8 and add 6. Someone must tell you that to add six you count to the right six times. Starting with eight, they will think or say as they count, "one, two, three, four, five, six," but they will be pointing to the numbers 9, 10, 11, 12, 13 and 15. They have thus effortlessly passed into the teens by following the rule of addition. While they may not be ready to add double digits, they should understand that adding six to anything requires them to begin at the starting number and count to the right six numbers.

When you begin to teach double digits (or triple), there is another concept your students must learn. Again, it is easy and intuitive with Numdrum, and there is no visual barrier when you cross into the next decade.

Choose the number 28, for example. Let's add 10 to that number. First, starting at 28, count to the right ten numbers. When the students stop at 38, make sure they understand that it is one column above the 28, where they started. 28, 38, 48, all go up 10 at a time.

Now let's add 40 to our starting number of 28. How do we do that? The left number tells you how many columns to move up—four. At the fourth column above 28 the student will find 68. They should understand that 4 + 2 = 6, even when they are in the tens column. With this visual confirmation, such problems are easy.

Another easy concept to teach with Numdrum is crossing decades. Starting with 28 again, let's add 34. Your students should tell you that you would add 30 by counting up three columns. Then you would add the four by counting to the right four more numbers. When they correctly find first 58, then 62, they will have successfully passed another major hurdle with ease.

Of course, first graders will not fully grasp the concepts without a lot of practice drill. Numdrum makes that a fun thing to do.

Problems that will not exist when you use Numdrum will be the transitions from single to multiple digits, from double to triple digits, and conflicts between up and down.

Before you can work with long addition, your children must be able to correctly add up to 10 + 9 and subtract 11 from any number in the range of 11 to 19.

Subtraction

After children learn addition, subtraction is easy with Numdrum. Children must simply understand that to add, you count up the columns or count up to the right; to subtract, you count down the columns or count down to the left. Again, with practice, students will learn quickly and without trauma to subtract numbers that pass from one decade to another.

When children can subtract a number from 10, they are ready to move to larger problems.

Multiplication

Learning the multiplication tables with Numdrum is equally fascinating for children. In the beginning, of course, we add 3 to 3, then to 6, then to 9. When we teach multiplication, however, we are going to mark the numbers with a dry erase marker. Mark the 3, add 3 and mark the 6. Mark the 6, add three and mark the 9. When you have completed the table for 3, you will have highlighted the numbers 3, 6, 9, 12, 15, 18, 21, 24 and 27. The visual confirmation will permit you to then count up by threes. If you start at 3 and count each mark to the right, the second mark is 6, the third is 9, the fourth is 12, and so forth.

Before you leave the table of three, consider pointing out that when the numbers reach 30, the digit sequence begins again—33, 36, 39, 42, 45, etc. In fact, any concept you can teach now, using Numdrum, will make later instruction easier.

Some multiplication tables will exhibit interesting patterns on Numdrum that can open up new realms of thought for your students to prepare them for higher mathematics. For instance, the table on 9 will illustrate that the numbers progress up one column and left one digit: 9, 18, 27, 36, etc. This is another important concept for children to learn, and again they learn it while doing something else.

People who work with numbers often transpose them—that is, look at 74 and write 47. This causes an error that is often very difficult to locate, since both numbers are present visually, but the result is different. Accountants know when they are looking for a transposed number. When an account is out of balance, an accountant will typically subtract the two numbers that don't agree to find out how far off the numbers are. In this case, 74 – 47 is 27. Since 27 is evenly divisible by 9, we can be certain that two numbers were transposed. Any number evenly divisible by 9 is the accountant's clue.

When your students learn that the table of nines involves these complimentary numbers, they have taken a step to understanding how nines work. How nines work will become very important later, when we learn to check our problems for accuracy. If you are a math teacher, you may be familiar with the technique called "casting out nines." If not, read on to find out how this is used.

Visually, the table of 8 also gives us some clues. 8, 16, 24, 32, 40, 48, 56, 64, 72, 80. The tens still go up one column, for the most part, but the digits drop by two each time. Visually, Numdrum will show the same digits four columns higher—8 and 48, 32 and 72. Adding by eight or counting by eight becomes easy when a student grasps this concept.

The tables of 2, 4, 5, 8 and 9 provide patterns that are visually easy to confirm with Numdrum. Tables for 3, 6 and 7 are more difficult to “see.” For 7, however, it is still easy enough to count the “times” 7 has been added to reach the marked number. Count 2, you’re at 14. Count 3, you’re at 21—and you remind your students that 7 times 3 is the same as 3 times 7. When they have already learned the tables up to 6 or 7, it is only the higher numbers that are a problem—6 times 6, 7, 8 and 9, 7 times 7, 8 and 9.

Tables of 11 and 12 again provide interesting patterns that visually help students learn them. Beyond that, we are moving into long multiplication and division, where you should learn to follow the rules given later in this book.

The following sections are based on Trachtenberg, Vedic or my dad’s “head math” concepts. Whenever children are ready to perform long addition is the time to teach it. I cannot tell you when that will be, because each student and each classroom is unique. If a student has used Numring in pre-school, he may be ready to do long addition midway through first grade. If none of the Numdrum series has been used by first grade, students may not be ready to do long addition until midway through second grade.

Many other activities and games can be performed or enhanced by using the Numdrum as a visual aid. Please visit John Harrison’s web site at www.numdrum.com for more information, or to order a set of Numdrums for your classroom. You and your students will be glad you did.

BASIC MATH,
GRADES 2 AND UP

As a child, even into my teen years, I was constantly amazed at how well my dad could do math in his head. He would attempt to explain the steps to me, but they were beyond my comprehension. To pass the time on a long drive, for instance, he would calculate something in his head. There were no car radios, CDs or cell phones in those days. We created our own distractions.

A typical example I remember well was, if you had, say, $300,000 in the bank and it earned 5½ percent interest, how much money would you be able to spend every day without touching the principal? Can you do that in your head? Another one was, if I finance something—a house, a car, a refrigerator—how much will I pay in interest over the course of a year's time? What will the actual cost be after 5 years, or 20 years, of making payments?

There were practical applications, too. Dad traveled in his work. He would compare the real cost of a new car, weighing the differences among various options—amount of down payment, trade-in amounts, or the slight variations in interest amounts that went with each option. Before the salesman could do it on paper, my father would know what the "real" cost of that car would be, and he would have made an informed decision about his purchase options. He did have an advantage here, because these were days long before the luxury of calculators, and the salesman had to do his figures on paper.

But are you ready for this? My dad did not graduate from high school. In fact, he left school in grade 10. As the eldest of six children, he had to quit school to help earn a living for the family. He had thus learned this "head math" before he reached grade 10. Maybe that makes you feel a little inadequate, especially if you're a math teacher or a college student who can't do that kind of math in your head. Here's news: you are smart enough; it's the educational system that has let you down big time, not your brain. The good news is, it's not too late to change.

Today there are several so-called "new" systems—lattice math, Vedic math, Kumon math—that all seem to bear a strange resemblance to one of the earlier methods. I know that neither my dad nor Jakow Trachtenberg used lattice, Vedic or Kumon math, so I approach these so-called "new" systems with hefty skepticism. Like the reading "systems," the new methods seem to be more about making money and selling books than they about teaching children.

What follows is a blending of styles and a simplification of the basic mathematics process. You do not need four ways to multiply, but one. Instead of presenting all the options, as other systems have done, I have selected just one way that works all the time with a minimum of exceptions. That is the approach that was taken in my dad's head math book. Later on, you can learn the “short cuts” and tricks if you wish. First you need to find, and master, one technique that always works.

The new systems all seem to have a need to “explain” how their methods work by providing complex examples in algebra to explain their approach. Dad’s book didn’t do that. Neither will this one. What you will get below is enough information to understand and apply the rules, and several examples. Again, you can learn the rest later, if you wish. It has no place in early childhood education.

The examples I have provided will not make you a math whiz. Both you and your students will require many such numbers if you are to achieve any success at speed or head math. Drill with your own problems. I give you the knowledge of 1) how to solve long or large math problems, 2) examples that include all the exceptions you will encounter, and 3) a way to check your work for accuracy. Armed with this information, you can make up your own problems. You are allowed to use a calculator, but only to check your work. Otherwise, work to do as much in your mind as you can remember.

You may realize immediately the value of using Numdrum early and then this system for basic arithmetic, as your students outpace their peers. Later, as you follow their progress, you can expect your students, whose early experience with numbers was positive and successful, to forge ahead when faced with new challenges, because you gave them the confidence they needed to do that. Either way, you will be rewarded and pleasantly surprised by your choice to teach arithmetic this way.

Add columns of any size numbers

Without ever adding more than 9 + 9 you will soon be able to add any list of numbers very fast. Some people find the answers as fast as another person can enter them into a calculator. First we will learn the rules with some easy problems. Later on we'll describe how you should think when you tackle addition problems. In the end, you will be able to do speed addition problems in your head.

If you already know long addition, you will understand how the rules work—almost. The fact that this system differs from what you know is what makes it work better. How you think as you work on a problem will increase your speed and accuracy. It will also permit you the luxury of gradually reducing your notes, or workspace as you commit more and more of the process to your memory. The rules, then, are primarily for those who are just starting out, until we get to the "head math" stage.

We will be adding one column at a time. We will begin with the column to the right. For teaching purposes only, we will label our columns. As we teach each rule, the part of the problem it refers to will be printed in bold red typeface.

RULES

Start by adding any carry from the column to the right.

When you reach or pass 11:

> Put one tick (`) in front of that number. (Later, you may count ticks on fingers.)
> Remove the tens digit from your count.
> Reduce the ones digit by one. (The net result of these two: subtract 11.)

To total each column, add:

> The ones digit.
> The ticks from that column.
> Ticks from the column to the right.

Now let's see how that works. We will start with easy problems until we understand how the rules apply. Then we will add more complex problems. Third, we will show you how to "think" as you work on a problem. Finally, we will show you a way to verify that you have a correct answer.

The following problems will help you to understand how the rules work. In the beginning the rules will seem to be holding you back, but soon you will discover that they help you to keep things straight in your mind. The rules are thus structured to help you achieve the mental agility you need to accomplish head math.

A	B
	5
	5

You know the answer to some of these easy problems without following the rules, but let's see how they work. 5 + 5 is 10. We did not reach or pass 11, so the first part of the rules are not required. We did, however, reach the bottom of the column, so we must total it. Write the units digit (0) and carry the tens digit (1) to the next column.

A	B
	5
	5
1	`0

It is important to understand that we are finished with column B, on the right. Column A has a zero in it, so when we add the carry to it, the total is 1. That completes this problem; the sum is 10.

See how the rules work with minor variations.

A	B
	5
	`6

5 + 6 = 11. Because we have reached 11, the rules require us to add a tick (`) to the number 6; remove the tens digit, leaving 1; then reduce the units digit by 1, leaving zero (0). (You could add a tick and subtract 11 if that's an easier rule to follow.) From the problem above, you know, of course, that zero is not the correct number for that column, but now we are going to follow a rule to total that column. In this case, the rule may seem more difficult that it needs to be, but bear with me; it is this very rule that will make it possible for you to soon increase your speed and commit to memory the steps of a problem.

A	B
	5
	`6
	0
	1
1	1

To total column B, add the tick to the zero. The final answer will be one (1).

As an older person, it may be difficult for you to switch your thoughts to these new rules; again, bear with me and you will soon understand the wisdom of this system. You expect to add this problem in the same way you added the previous one; 5 + 6 = 11, write the units (1) in the right column and carry the tens (1) to the next column. A long column of numbers can become very cumbersome, however. By following this new rule now, you will make it much easier to do columns of any size numbers and there will be no exceptions to the rules to confuse either you or your students.

Okay, I'm done preaching. We have a total for the column on the right, zero (0) + 1 tick = 1. Write the 1. There is nothing to carry to the next column.

To find the sum for column A, add the zero (0) and the one (1) tick from the column to the right. There was nothing to carry this time. Final sum is 11. We have achieved the correct answer by a slight adjustment of the familiar rules.

A	B
	6
	`6

One more variation. This time, 6 + 6 = 12. We passed 11, so add a tick, remove the tens digit (1) and reduce the ones digit (2 – 1 = 1). We are again ready to total the column. 1 + 1 dot = 2. There are no columns to the right of this one, so write the 2 and we are finished with column B.

A	B
	6
	`6
	1
	1
1	2

Again there is no carry, and column A contains nothing; add the tick from the column to the right and that total is 1. Write it below that column.

Now let’s increase the difficulty slightly.

A	B
	3
	`9
	8

3 + 9 = 12. We passed 11, so add a tick, remove the tens digit (1) and reduce the ones digit (2 – 1 = 1). Since we have not reached the bottom of the column, we continue with one (1) as our starting point. 1 + 8 = 9. To total the column, we add one tick below the 9. The final column total is 10. Write the zero (0) under column B and carry the one (1) to column A. Starting with the carried one (1), we add the single tick from column B, on the right, for a total of 2. Write the two (2) under column A.

A	B
	3
	`9
	8
1	9
	1
2	`0

A	B
	4
	`9
	`9

4 + 9 = 13. We passed 11. Add tick, remove tens digit (1), reduce ones digit (3 – 1 = 2) and continue. 2 + 9 = 11. Add tick, reduce 1 to zero (0). Ready to total the column. Zero (0) plus two (2) ticks = 2. No column to the right, so 2 is the correct final total for column B. Nothing to carry this time. For column A, start with zero (0). 0 + two (2) ticks from column B, to the right, gives a sum of 2. Write that number below column A. The sum of 4 + 9 + 9 is 22.

A	B
	4
	`9
	`9
	0
	2
2	2

With these few rules in place, it is time to look at some long addition problems. Long columns of numbers may be added by using the same rules. In the process, you will see how the rules work to keep any problem of addition as easy as these first examples have been.

Example 1

Beginning with this example, the description of the process will be simplified somewhat and the process for each column will have its own graphic display.

A	B		**Column B**
	7		
	\`5		7 + 5 = 12. Tick, reduce to 1.
	8		1 + 8 = 9.
	\`2		9 + 2 = 11. Tick, reduce to zero (0).
	6		0 + 6 = 6.
	3		6 + 3 = 9.
	9		Total:
	2		9 + 2 ticks = 11. Write 1, carry 1
	\`1		

A	B		**Column A**
	7		Carry (1) + two (2) ticks from right column = 3.
	\`5		Write 3.
	8		
	\`2		
	6		
	3		
1	9		
	2		
3	1		

The next examples will add more columns.

Example 2

A	B	C
	9	**3**
	1	**5**
	5	**`7**
		4
		1
		5

Column C

3 + 5 = 8.
8 + 7 = 15. Tick, reduce to 4

Total:
4 + 1 tick = 5.

A	B	C
	9	3
	1	5
	`5	`7
	4	4
	1	**1**
	6	5

Column B
9 + 1 = 10
10 + 5 = 15. Tick, reduce to 4.

Total:
4 + 1 tick from this column = 5.
5 + 1 tick from the column to the right = 6.

A	B	C
	9	3
	1	5
	`5	7
	4	4
	1	1
1	6	5

Column A

Total:
1 tick from the column to the right = 1.

Example 3

A	B	C	**D**
	4	8	**7**
	6	9	**3**
	5	0	**`9**
	3	2	**`6**
			3
			2
			5

Column D

7 + 3 = 10.
10 + 9 = 19. Tick, reduce to 8.
8 + 6 = 14. Tick, reduce to 3.

Total:
3 + 2 ticks = 5

A	B	**C**	D
	4	**8**	7
	6	**`9**	3
	5	**0**	`9
	3	**2**	`6
		8	3
		1	**2**
		`1	5

Column C

8 + 9 = 17. Tick, reduce to 6**.**
6 + 0 = 6.
6 + 2 = 8.

Total:
8 + 1 tick = 9.
9 + 2 ticks right = 11. Write 1, carry 1

A	**B**	C	D
	4	8	7
	`6	9	3
	5	0	`9
	3	2	`6
	8	8	3
	1	**1**	2
	`0	1	5

Column B
4 + carried 1 = 5.
5 + 6 = 11. Tick, reduce to 0.
0 + 5 = 5.
5 + 3 = 8.

Total:
8 + 1 tick = 9.
9 + 1 tick to the right = 10.
Write 0, carry 1

A	B	C	D
	4	8	7
	`6	9	3
	5	0	`9
	3	2	`6
1	8	8	3
	1	1	2
2	0	1	5

Column A

Carried 1

Total:
Carried 1 + 1 tick to the right = 2.
Write 2.

Example 4

Now add a section or the right to show how to think: finish something, then forget it.

A	B	C	**D**	**Column D**	**Think...**
	3	4	**5**		5
	6	2	**`7**	5 + 7 = 12. Tick, reduce to 1.	12, tick, 1
	1	3	**8**	1 + 8 = 9.	9
3	9	2	**`4**	9 + 4 = 13. Tick, reduce to 2.	13, tick, 2
		3	**7**	2 + 7 = 9	9
			9		
			2	**Total:**	1
			`1	9 + 2 ticks = 11. Write 1, carry 1.	Carry 1

A	B	**C**	D	**Column C**	
	3	**4**	5	4 + carry = 5.	5
	6	**2**	`7	5 + 2 = 7.	7
	1	**3**	8	7 + 3 = 10.	10
3	9	**`2**	`4	10 + 2 = 12. Tick, reduce to 1.	12. tick, 11
		3	7	1 + 3 = 4.	4
		4	9	**Total:**	
		1	**2**	4 + 1 tick = 5.	5
		7	1	5 + 2 right ticks = 7.	7

A	**B**	C	D	**Column B**	
	3	4	5		
	6	2	`7	3 + 6 = 9.	9
	1	3	8	9 + 1 = 10.	10
3	**`9**	`2	`4	10 + 9 = 19. Tick, reduce to 8.	19, tick, 8
		3	7		
	8	4	9	**Total:**	
	1	**1**	2	8 + 1 tick = 9.	9
	`0	7	1	9 + 1 tick right = 10. Write 0, carry 1.	10, zero, carry 1

A	B	C	D	**Column A**	
	3	4	5		
	6	2	`7		
	1	3	8		
3	`9	`2	`4	3 + carried 1 = 4	4
		3	7		
4	8	4	9		
	1	1	2	**Total:**	
5	0	7	1	4 + 1 tick right = 5.	5

Example 5

A	B	C	**D**
	5	2	**3**
	6	4	**`8**
	9	5	**2**
	6	3	**1**
			3
			1
			4

Column D

3 + 8 = 11. Tick, reduce to 0.
0 + 2 = 2.
2 + 1 = 3.

Total:
3 + 1 tick = 4. Write 4.

Think...

11, tick, 0
2
3

4

A	B	**C**	D
	5	**2**	3
	6	**4**	`8
	9	**`5**	2
	6	**3**	1
		3	3
		1	**1**
		5	4

Column C

2 + 4 = 6.
6 + 5 = 11. Tick, reduce to 0.
0 + 3 = 3.

Total:
3 + tick = 4.
4 + 1 tick right = 5. Write 5

6
11, tick, 0
3

4

5

A	**B**	C	D
	5	2	3
	`6	4	`8
	9	`5	2
	`6	3	1
	4	3	3
	2	**1**	1
	7	5	4

Column B

5 + 6 = 11. Tick, reduce to 0.
0 + 9 = 9.
9 + 6 = 15. Tick, reduce to 4.

Total:
4 + 2 ticks = 6.
6 + 1 tick right = 7.

11, tick, 0
9
15, tick, 4

6. 7

A	B	C	D
	5	2	3
	`6	4	`8
	9	`5	2
	`6	3	1
	4	3	3
	2	1	1
2	7	5	4

Column A

Total:
2 right ticks = 2

2

Example 6

For our final example of long column addition, let's get extreme, just to lock into your mind how easy this system is compared to the traditional way you do math.

A	B	C	Column C	Think...
	7	**9**		9
	5	**`8**	9 + 8 = 17. Tick, reduce to 6.	17, tick, 6
	6	**`9**	6 + 9 = 15. Tick, reduce to 4.	15, tick, 4
	2	**`7**	4 + 7 = 11. Tick, reduce to 0.	11, tick, 0
	3	**6**	0 + 6 = 6.	6
	5	**`8**	6 + 8 = 14. Tick, reduce to 3.	14. tick, 3
	9	**7**	3 + 7 = 10.	10
	9	**`5**	10 + 5 = 15. Tick, reduce to 4.	15, tick, 4
	7	**4**	4 + 4 = 8.	8
	7	**`7**	8 + 7 = 15. Tick, reduce to 4.	15, tick, 4
	5	**`7**	4 + 7 = 11. Tick, reduce to 0.	11. tick, 0
	9	**5**	0 + 5 = 5.	5
	5	**3**	5 + 3 = 8.	8
	8	**`5**	8 + 5 = 13. Tick, reduce to 2.	13, tick, 2
	4	**2**	2 + 2 = 4.	4
	5	**`9**	4 + 9 = 13. Tick, reduce to 2.	13, tick, 2
	9	**`9**	2 + 9 = 11. Tick, reduce to 0.	11, tick, 0
	7	**5**	0 + 5 = 5.	5
	6	**4**	5 + 4 = 9.	9
	8	**`3**	9 + 3 = 12. Tick, reduce to 1.	12, tick, 1
	9	**5**	1 + 5 = 6.	6
	3	**1**	6 + 1 = 7.	7
	8	**`8**	7 + 8 = 15. Tick, reduce to 4.	15, tick, 4
	7	**`9**	4 + 9 = 13. Tick, reduce to 2.	13, tick, 2
	9	**7**	2 + 7 = 9.	9
	8	**`9**	9 + 9 = 18. Tick, reduce to 7.	18, tick, 7
	6	**`6**	7 + 6 = 13. Tick. reduce to 2.	13, tick, 2
	4	**7**	2 + 7 = 9.	9
	8	**`9**	9 + 9 = 18. Tick, reduce to 7.	18, tick, 7
	6	**`8**	7 + 8 = 15. Tick, reduce to 4.	15, tick, 4
	5	**4**	4 + 4 = 8.	8
	8	**`8**	8 + 8 = 16. Tick, reduce to 5.	16, tick, 5
	7	**4**	5 + 4 = 9.	9
	5	**`6**	9 + 6 = 15. Tick, reduce to 4.	15, tick, 4
	9	**`8**	4 + 8 = 12. Tick, reduce to 1.	12, tick, 1
	7	**9**	1 + 9 = 10.	10
		10	**Total:**	
		20	10 + 20 ticks = 30. Write 0, carry 3	20, 30, 0 carry 3
		```0		

A	B	C	Column B	Think...
	7	9	Carry 3 + 7 = 10	10
	`5	`8	10 + 5 = 15. Tick, reduce to 4.	15, tick, 4
	6	`9	4 + 6 = 10.	10
	`2	`7	10 + 2 = 12. Tick, reduce to 1.	12, tick, 1
	3	6	1 + 3 = 4.	4
	5	`8	4 + 5 = 9.	9
	`9	7	9 + 9 = 18. Tick, reduce to 7.	18, tick, 7
	`9	`5	7 + 9 = 16. Tick, reduce to 5.	16, tick, 5
	`7	4	5 + 7 = 12. Tick, reduce to 1.	12, tick, 1
	7	`7	1 + 7 = 8.	8
	`5	`7	8 + 5 = 13. Tick, reduce to 2.	13, tick, 2
	`9	5	2 + 9 = 11. Tick, reduce to 0.	11, tick, 0
	5	3	0 + 5 = 5.	5
	`8	`5	5 + 8 = 13. Tick, reduce to 2.	13, tick, 2
	4	2	2 + 4 = 6.	6
	`5	`9	6 + 5 = 11. Tick, reduce to 0.	11, tick, 0
	9	`9	0 + 9 = 9.	9
	`7	5	9 + 7 = 16. Tick, reduce to 5.	16, tick, 5
	`6	4	5 + 6 = 11. Tick, reduce to 0.	11, tick, 0
	8	`3	0 + 8 = 8.	8
	`9	5	8 + 9 = 17. Tick, reduce to 6.	17, tick, 6
	3	1	6 + 3 = 9.	9
	`8	`8	9 + 8 = 17. Tick, reduce to 6.	17, tick, 6
	`7	`9	6 + 7 = 13. Tick, reduce to 2.	13, tick, 2
	`9	7	2 + 9 = 11. Tick, reduce to 0.	11, tick, 0
	8	`9	0 + 8 = 8.	8
	`6	`6	8 + 6 = 14. Tick, reduce to 3.	14, tick, 3
	4	7	3 + 4 = 7.	7
	`8	`9	7 + 8 = 15. Tick, reduce to 4.	15, tick, 4
	6	`8	4 + 6 = 10.	10
	`5	4	10 + 5 = 15. Tick, reduce to 4.	15, tick, 4
	`8	`8	4 + 8 = 12. Tick, reduce to 1.	12, tick, 1
	7	4	1 + 7 = 8.	8
	`5	`6	8 + 5 = 13. Tick, reduce to 2.	13, tick, 2
	`9	`8	2 + 9 = 11. Tick, reduce to 0.	11, tick, 0
	7	9	0 + 7 = 7.	7
4	7	10	**Total Column B**:	
	21	20	7 + 21 ticks + 20 right ticks = 48.	28, 48. 8, carry 4
25	````8	```0	Write 8, carry 4.	
			Total Column A:	4, 25
			4 + 21 right ticks = 25. Write 25.	

Final sum = 2580

To add Column C the traditional way, you must carry 23. In our system, you only carried 3. If you had added Column B to that 23 you carried, by the time you reached the bottom of the second column, you would need to add as high as 258. For our system, we carried 4. Why handle so much extra baggage when you can make life simple with this system? The baggage is especially difficult for small children. They will feel much more confident about math when they can be successful at this beginning level. We owe it to our children to change our basic math paradigm.

People still make mistakes, even if they are simply entering numbers into a calculator or computer. It is a good idea to check your work. If you can find such errors, you will be ahead of the game.

Now let’s learn a fast and simple way to check your addition for accuracy.

Checking your work

A	B	C	D
	5	2	3
	`6	~~4~~	~~`8~~
	~~9~~	~~`5~~	2
	`6	3	~~1~~
	4	3	3
	2	1	1
2	7	5	4

To illustrate how to check the accuracy of an addition problem, we will use example 5. As before, we will work with one column at a time to find a check-sum for that column.

First, cross out all number combinations that add up to nine (9). (This is the "casting out nines" technique I mentioned earlier.) In column D, that would be the 8 and the one (1). Now add the remaining numbers: 3 + 2 = 5. Put that number below the column.

For column C, numbers that add to 9 are 4 and 5. We cross them out. Adding the others, we get 2 + 3 = 5.

For column B, only the number 9 gets crossed out. Adding 5 + 6 + 6, we get 17. But here we do another step. We add the tens digit (1) to the units digit (7) for a single digit result, and write the sum, 8, under that column. We suggest that you "reduce" to a single digit as you go. It works like this: 5 + 6 = 11. Reduce (1 + 1) to 2. 2 + 6 = 8.

The numbers under the columns are now 8, 5 and 5.

Using the work area, we clone the tick line to get the following:

A	B	C	D
	4	3	3
	2	1	1
	2	1	1
	8	5	5

Then we add the columns to get the total. The total for each column should match the column total of the problem: 8, 5 and 5. If one column does not match, that is the column with the error. You only need to correct that column in the problem.

Finally, we are going to get a check-sum for the 855 and the problem sum, 2754, by adding across and reducing to a single digit as we go. We'll start with the 855.

8 + 5 = 13. Reducing, 1 + 3 = 4. 4 + 5 = 9. Our check-sum digit is 9. This is to show you that either way will work, but reducing as we go is easier, especially for long columns. So we'll do it over to show you the check-sum will still be the same number.

8 + 5 = 13. 13 + 5 = 18. Then reducing to a single digit, 1 + 8 = 9.

Now for some real magic. Do the same thing for the sum of the problem, 2754.

2 + 7 = 9. Casting out those two numbers, we have 5 + 4 = 9. Same check-sum.

Example 5 is correct. If any of these check-sums had not matched, the error would have been in the column without the match. When you make an error using this system, you need only to correct the column with the error.

For a review, consider what we have accomplished.

1. First and foremost, you or your student can quickly achieve success. For a child, that means a boost in self-esteem and a confidence in his ability to solve problems.
2. We have an easy, quick way to add columns of numbers, long or short.
3. We have an easy way to check our addition to make sure it is correct.
4. We have an easy, one-column fix if we find an error.
5. Last, and perhaps most important, we have a way to think about adding. It will permit you to gradually do more and more basic math in your head and amaze your friends who don't know this system.

At this point, you should practice doing problems of addition. With practice, you should be able to do any long addition problem in the time it takes someone to enter the numbers into a computer or calculator.

Subtraction, multiplication and division are also easy to learn and fast to do. We will learn how to do subtraction next.

Subtraction

RULE: Changing both numbers by the same amount (+ or -) will not change the answer.

HINT: The idea is to round the bottom number up or down to even 10's, 100's or 1000's, until you have nothing but zero to subtract.

Again, we will begin with some very elementary problems to illustrate how the rule works. It is critical that you learn to do this step in your mind before we go forward with more difficult subtraction problems, because every example (and most problems) will require you to perform this step.

Example 1

$$\begin{array}{r} 14 \\ -8 \\ \hline 6 \end{array}$$

If we add 2 to both numbers, we get:

$$\begin{array}{rcrcr} 14 & + & 2 & = & 16 \\ 8 & + & 2 & = & -10 \\ \cline{5-5} & & & & 6 \end{array}$$

Example 2

$$\begin{array}{r} 47 \\ -29 \\ \hline 18 \end{array}$$

If we add 1 to both numbers, we get:

$$\begin{array}{rcrcr} 47 & + & 1 & = & 48 \\ -29 & + & 1 & = & -30 \\ \cline{5-5} & & & & 18 \end{array}$$

Example 3

For this example, we will round the bottom number a second time. First to the nearest tens, then to the nearest 100's..

$$\begin{array}{r} 136 \\ -67 \\ \hline 69 \end{array}$$

$$\begin{array}{rcrcr} 136 & + & 3 & = & 139 \\ -67 & + & 3 & = & -70 \end{array}$$

$$\begin{array}{rcrcr} 139 & + & 30 & = & 169 \\ 70 & + & 30 & = & -100 \\ \cline{5-5} & & & & 69 \end{array}$$

You could do larger problems by repeating these steps many times, but there is an easier and quicker way to subtract large numbers. Just round up each column to the nearest 10.

Using the technique of rounding up and the ticks rule we learned for doing long addition, we can easily subtract one large number from another and get the answer very fast.

First we learn the rules with a few examples. Then we will train you how to think as you attack a problem. Soon you will be able to do such problems in your head and just put the correct answer on paper.

RULES:

We work from right to left, as we did in addition problems.

1. If there is a borrow, start by reducing the top number by one.

2. Subtract the column. If the bottom number is higher...
   a. Round the bottom number up to 10.
   b. Add the same amount to the top number.
   c. Subtract and mark a tick on the left column for the borrow.

3. Write the answer and go to the next column to the left.

Example 1

B	**A**	**Column A**
9	**\`2**	4 is larger than 2. Round it up to 10. 4 + 6 = 10.
-3	**4**	Add 6 to the top number also: 2 + 6 = 8. Write this as the answer for column A.
	8	Put a tick with the 2, because we "borrowed" a number.

HINT: Each time you round up to 10, the sum at the top number is also the answer.

2 – 4 is really 12 – 4. That's why we borrow one from the next column to the left.
When we add 6 to both numbers, we get 18 – 10, or 8.

Other examples:

13	+	3	=	16
-7	+	3	=	-10
				6

15	+	1	=	16
-9	+	1	=	-10
				6

11	+	6	=	17
-4	+	6	=	-10
				7

B	A	**Column B**
9	\`2	Begin by reducing 9 to 8 because of the tick we borrowed from column A.
-3	4	8 – 3 = 5. This is the answer for column B.
5	8	

Now let’s look at some more complicated problems. Well, once they were very complicated. Now they are very easy.

Example 2

A	B	C	**D**
1	4	2	**`3**
	7	9	**6**
			7

Column D

Round 6 up to 10 by adding 4.
Think: tick 3 + 4 = 7.

A	B	**C**	D
1	4	**`2**	`3
	7	**9**	6
		2	7

Column C

Reduce 2 to 1 for the tick (borrow) at column D.
Round 9 to 10 by adding 1.
Think: tick, 1 + 1 = 2.

A	**B**	C	D
1	**`4**	`2	`3
	7	9	6
	6	2	7

Column B

Reduce 4 to 3 and round 7 up to 10 by adding 3.
Think: tick, 3 + 3 = 6.

A	B	C	D
1	`4	2	`3
	7	9	6
	6	2	7

Column A

Reduce 1 to 0. Nothing to subtract now, so we’re finished.

Got the idea? Let’s try a larger problem. No notes this time, just the problem the way it would appear on your paper when you do all the calculations in your head.

Example 3

This looks hard, doesn't it? Just read left to right and follow the rules; you'll do fine.

4	2	3	4	8	5	9	2	1	7	**`3**
	8	3	4	2	5	7	5	8	4	**7**
										6

4	2	3	4	8	5	9	2	1	**7**	`3
	8	3	4	2	5	7	5	8	**4**	7
									2	6

4	2	3	4	8	5	9	2	**`1**	7	3
	8	3	4	2	5	7	5	**8**	4	7
								3	2	6

4	2	3	4	8	5	9	**`2**	`1	7	`3
	8	3	4	2	5	7	**5**	8	4	7
							6	3	2	6

4	2	3	4	8	5	**9**	`2	`1	7	`3
	8	3	4	2	5	**7**	5	8	4	7
						1	6	3	2	6

4	2	3	4	8	**5**	9	`2	`1	7	`3
	8	3	4	2	**5**	7	5	8	4	7
					0	1	6	3	2	6

4	2	3	4	**8**	5	9	`2	`1	7	`3
	8	3	4	**2**	5	7	5	8	4	7
				6	0	1	6	3	2	6

4	2	3	**4**	8	5	9	`2	`1	7	`3
	8	3	**4**	2	5	7	5	8	4	7
			0	6	0	1	6	3	2	6

4	2	**3**	4	8	5	9	`2	`1	7	`3
	8	**3**	4	2	5	7	5	8	4	7
		0	0	6	0	1	6	3	2	6

4	**`2**	3	4	8	5	9	`2	`1	7	`3
	8	3	4	2	5	7	5	8	4	7
	4	0	0	6	0	1	6	3	2	6

4	`2	3	4	8	5	9	`2	`1	7	`3
	8	3	4	2	5	7	5	8	4	7
3	4	0	0	6	0	1	6	3	2	6

This problem is too long for a calculator. It is even difficult for a computer. But it is easy for you to do, isn't it?

Multiplication

The process of multiplication will require three areas on your work paper. First is the area where you will present the problem. We will again begin with an easy problem and progress to more difficult ones, to make it easy to grasp the rules.

To help us visualize our problem and its process, we will be using a single line display. The multiplicand is shown first, then an "x" to signify the multiplication operation, then the multiplier. Below are two examples, one with two digits in each and one with three digits in each.

1	2	x	3	4		1	2	3	x	4	5	6
A	B		C	D		A	B	C		D	E	F

The second area will be an answer space. Reserve a column for each digit of the multiplicand and each digit of the multiplier. In the first sample above, we would have four columns for the answer. In the second sample, we need six columns.

The answer area will have three rows for figures. The first row we will call "ones." It will hold the unit amounts from the process of multiplication. The second row we will call "tens." It will be used if a product requires us to carry a tens digit to the next column. The third row is where we put our final answer. To begin, we will name the rows. Later, when you know what each row represents, the names will be dropped, along with the letters that identify columns.

	A	B	C	D
Ones				
Tens				
Answer				

The answer area should be just below the area where you present your problem. That will leave the area to the right and at the bottom of the page for your workspace.

The workspace is special because when you are learning how to perform multiplication, you will write out each step of the process. Later, as you become better able to follow the procedure without this help, some of the workspace will not be needed. Eventually our goal is to do everything in our head that is now written out in the workspace. To make this happen, though, you must be careful during this early stage to learn the best way to think about the process. It is the thinking part that will make you work faster and more accurately.

Are you ready to begin? Let's work with a two-digit multiplicand and a two-digit multiplier until you understand the rules.

Example 1

1	**2**	x	3	**4**
A	**B**		C	**D**

Present your problem and set up your answer area below the problem display.

	A	B	C	**D**
Ones				**8**
Tens				
Answer				

Our problem area and our answer area will highlight the portion of the problem we are working on, to help you become familiar with the process. The simplest way to state the rule for multiplication is this:

1. Right
2. Outside to Inside and Add
3. Left.

We will explain what this means as we perform some examples.

The first rule is multiply right. That means the digit on the right of the multiplicand must be multiplied with the digit on the right of the multiplier. The numbers in red above show that column B is multiplied with column D. These ore the rightmost numbers.

Our workspace shows: 2 x 4 = 8. In your answer area, put an 8 on the ones line of column D.

The next rule is multiply from outside to inside and add. We are going to shift our focus to column A and place a bracket to show that column A represents the outside of the multiplicand and column D represents the outside of the multiplier.

1	2	x	3	**4**
A	B		C	**D**

Multiply the two outside numbers (A and D)
1 x 4 = 4

1	**2**	x	**3**	4
A	**B**		**C**	D

.Multiply the two inside numbers (B and C).
2 x 3 = 6

Add the products. 4 + 6 = 10. Our workspace should look like this:

1	x	4	=		**4**
2	x	3	=		**6**
				1	**0**

When we put this sum (10) in the answer area, we will put the zero (0) in the ones line under column C. This time we have a one (1) to carry. Put that in column C, on the "tens" line. Now your answer area will look like this:

	A	B	C	D
Ones			**0**	8
Tens		**1**		
Answer				

1	2	x	**3**	4
A	B		**C**	D

The last step is to multiply the numbers on the left, columns A and C

	A	B	C	D
Ones		**3**	0	8
Tens		1		
Answer				

1 x 3 = 3. Put this answer in the next ones column to the left, column B.

In some cases, as here, we will not need column A, because all of our multiplication is finished.

The final step is to add the ones and the tens columns for our answer

	A	B	C	D
Ones		3	0	8
Tens		1		
Answer		**4**	**0**	**8**

On paper, your problem should look like this. How much of the workspace could you do in your head? Even at this stage, you may be able to do it all in your head.

1	2	x	3	4
A	B		C	D

	A	B	C	D
Ones		3	0	8
Tens		1		
Answer		4	0	8

Right	2	x	4	=		8
Outside	1	x	4	=		4
Inside	2	x	3	=		+6
Add					1	0
Left	3	x	1	=		3

Example 2

One step at a time, the highlights will guide you through the process.

	A	**B**		C	**D**							
	7	**3**	x	6	**4**	**Right**	**3**	**x**	**4**	**=**	**1**	**2**
						Outside						
	E	F	**G**	**H**		Inside						
Ones				**2**		Add						
Tens			**1**									
Answer						Left						

	A	B		C	**D**							
	7	3	x	6	**4**	Right	3	x	4	=	1	2
	7	**3**	x	**6**	4							
						Outside	**7**	**x**	**4**	**=**	**2**	**8**
	E	F	G	H		**Inside**	**3**	**x**	**6**	**=**	**1**	**8**
Ones				2		Add						
Tens			1									
Answer						Left						

	A	B		C	D							
	7	3	x	6	4	Right	3	x	4	=	1	2
						Outside	7	x	4	=	**2**	**8**
						Inside	3	x	6	=	**1**	**`8**
						Add					**3**	**5**
	E	**F**	**G**	H								**1**
Ones			**6**	2							**4**	**6**
Tens		**4**	1									
Answer						Left						

	A	B		C	D							
	7	3	x	**6**	4	Right	3	x	4	=	1	2
						Outside	7	x	4	=	2	8
						Inside	3	x	6	=	1	`8
						Add					3	5
	E	F	G	H								1
Ones		**2**	6	2							4	6
Tens	**4**	4	1									
Answer	**4**	**6**	**7**	**2**		Left	**7**	**x**	**6**	**=**	**4**	**2**

The completed problem should look like this on paper:

	A	B		C	D							
	7	3	x	6	4	Right	3	x	4	=	1	2
						Outside	7	x	4	=	2	8
						Inside	3	x	6	=	1	`8
						Add					3	5
	E	F	G	H								1
Ones		2	6	2							4	6
Tens	4	4	1									
Answer	4	6	7	2		Left	7	x	6	=	4	2

As we add more digits to either the multiplicand or the multiplier, the process expands also. We still start with the right side and end with the left side, but the second rule expands to accommodate the new digits. Before we tackle a full-scale process, let’s look at how rule number two changes with each additional digit to handle.

At this point, we will also drop the column designations. To help us keep track of where we are in the process, we will place a mark above our starting point for each step. It is easy to follow for up to four digits. Beyond four digits, we will need the mark for sure.

Three digits, step 2:

	*					*
1	**2**	3	x	4	5	**6**

*						*
1	2	3	x	4	5	**6**

*					*	
1	2	3	x	4	**5**	6

	*					*
1	2	**3**	x	4	**5**	6

*						*
1	**2**	3	x	4	**5**	6

*					*	
1	**2**	3	x	**4**	5	6

*						*
1	2	**3**	x	**4**	5	6

The first section of step 2 is just like multiplying two-digit problems outside to inside. We would do 2 x 6 = 12, and 3 x 5 = 15, then add 12 + 15 = 27. The last section of step 2 is also the same as multiplying two-digit problems outside to inside. In this case, it would be 1 x 5 = 5, 2 x 4 = 8, and adding 8 + 5 = 13.

Notice that each number of the multiplicand must be multiplied with each number of the multiplier. When we have just two more digits to multiply, we must perform 7 sections in step two instead of two. The rules are the same, but the process does get more complicated, so it is very important to keep track of our positions by marking the outside numbers at each step.

Here, the second part of step two requires us to look at the 1 and the 6 as outside numbers for three digits. Our answer will multiply outside (1 x 6 = 6) middle (2 x 5 = 10) and inside (3 x 4 = 12)—hence “outside to inside.” Adding 6 + 10 + 12 = 28, the sum goes in the ones and tens columns of the answer.

Before we do more problems, let’s see how step two works when we add just one more digit to the multiplicand and one more to the multiplier.

Four digits, step 2:

2A = 2 digits

		*						*
1	2	**3**	4	x	5	6	7	**8**

		*						*
1	2	3	**4**	x	5	6	**7**	8

2B = 3 digits

	*							*
1	**2**	3	4	x	5	6	7	**8**

	*							*
1	2	**3**	4	x	5	6	**7**	8

	*							*
1	2	3	**4**	x	5	**6**	7	8

2C = 4 digits

*								*
1	2	3	4	x	5	6	7	**8**

*								*
1	**2**	3	4	x	5	6	**7**	8

*								*
1	2	**3**	4	x	5	**6**	7	8

*								*
1	2	3	**4**	x	**5**	6	7	8

2D = 3 digits

*							*	
1	2	3	4	x	5	6	**7**	8

*							*	
1	**2**	3	4	x	5	**6**	7	8

*							*	
1	2	**3**	4	x	**5**	6	7	8

2E = 2 digits

*						*		
1	2	3	4	x	5	**6**	7	8

*						*		
1	**2**	3	4	x	**5**	6	7	8

We added two more digits to the problem and now we have 14 sections to complete. Just remember that each section in step two follows the same rule: Multiply from outside to inside, then add.

Because adding digits to our multiplicand and our multiplier also adds extra sections to step 2, you will need much more working space to complete one problem. It is important to remember where you are, and also to organize your workspace. The following problems, with three, four and five digits to multiply, show one way to organize your workspace.

The entire workspace should not be required, even now. If you can multiply 2 x 5, for instance, and put 10 in the list of numbers to be added, you do not need to display the multiplicand and multiplier in your workspace. Then each section of step 2, for instance, would have from 2 to 5 numbers to add.

Example 3

We have displayed the problem five times, one for each section. Your problem would only need to display it once. As you perform each section and move to the next, you can erase the previous mark or leave it. Either way, you will know what is the outside numbers you are currently multiplying.

Also note that when you reach the left digit of the multiplicand, then your mark stays at that digit and you move the mark on the outside digit of the multiplier until both marks are at the left side. Then you perform the left multiplication to complete the problem.

		*				*
3	**2**	**5**	**x**	**4**	**6**	**8**

Right	5	x	8	=	**4**	**0**

	*	*				*
3	2	5	x	4	6	8

2A	Outside	2	x	8	=	1	6
	Inside	5	x	6	=	3	0
	Add					**4**	**6**

*	*	*				*
3	2	5	x	4	6	8

2B	Outside	3	x	8	=	2	4
		2	x	6	=	1	2
	Inside	5	x	4	=	2	0
	Add					**5**	**6**

*	*	*			*	*
3	2	5	x	4	6	8

2C	Outside	3	x	6	=	1	8
	Inside	2	x	4	=		`8
	Add						5
							1
						2	**6**

*	*	*		*	*	*
3	2	5	x	4	6	8

Left	3	x	4	=	**1**	**2**

Ones		2	6	6	6	0
Tens	1	2	`5	`4	4	
(Add)	1	4	0	0	`0	0
			1	1		
Answer	1	5	2	1	0	0

For the previous problem, could you perform the multiplication steps in your mind and write only the products in your workspace? On the left below is the workspace we actually used. On the right is how it would look if you could multiply in your head the first parts of the problem.

	Right	5	x	8	=	**4**	**0**
2A	Outside	2	x	8	=	1	6
	Inside	5	x	6	=	3	0
	Add					**4**	**6**
2B	Outside	3	x	8	=	2	4
		2	x	6	=	1	2
	Inside	5	x	4	=	2	0
	Add					**5**	**6**
2C	Outside	3	x	6	=	1	8
	Inside	2	x	4	=		`8
	Add						5
							1
						2	**6**
	Left	3	x	4	=	**1**	**2**

Right	**4**	**0**
2A	1	6
	3	0
	4	**6**
2B	2	4
	1	2
	2	0
	5	**6**
2C	1	8
		`8
		5
		1
	2	**6**
Left	**1**	**2**

You could save some space by arranging your work area with columns of addition as shown below, from right to left below. For our next problem, we will display our workspace in this manner.

R		2A		2B		2C		L	
4	**0**	1	6	2	4	1	8	**1**	**2**
		3	0	1	2		`8		
		4	6	2	0		5		
				5	6		1		
		4	**6**			**2**	**6**		
				5	**6**				

Example 4

*	*	*	*		*	*	*	*
6	4	7	1	x	8	2	3	5

Ones		8	4	2	4	3	8	5
Tens	4	`4	`8	6	4	3		
(Add)	4	1	1	8	8	6	8	5
		1	1					
Answer	5	3	2	8	8	6	8	5

R	2A	2B	2C	2D	2E	L
5	3 5	2 0	3 0	1 8	1 2	4 8
	3	2 1	1 2	`8	3 2	
	3 8	2	1 4	5 `6	4 4	
		4 3	`8	6 0		
	3 8		5 3	2	4 4	
		4 3	1	8 2		
			6 4			

After you have mastered this form of workspace display, the next challenge is to also do the addition in your head. This requires you to remember more steps. In 2C, for instance, you would perform in your head 6 x 5 = 30, remember 30; 4 x 3 = 12, 30 + 12 = 42, remember 42; 7 x 2 = 14, 42 + 14 = 56, remember 56; and 1 x 8 = 8, 56 + 8 = 64. Put 4 in the ones line and 6 in the tens line below and to the left. As soon as you can do this much in your head, you can omit the entire workspace.

Here's a challenge for you. Children who learn the Trachtenberg system can multiply problems such as the one below in 70 seconds.[*] With practice, you can also accomplish this feat. The answer will have 22 digits, too many for your calculator, but not for you. The largest portion of step 2 will contain 10 products to add, so you will probably want to use a workspace to ensure accuracy.

5132437201 x 352736502785

Work your way up to this problem by making up your own for drills.

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[*] "The Trachtenberg Speed System of Basic Mathematics," translated and adapted by Ann Cutler and Rudolph McShane. 1960. Doubleday & Company, Inc., Garden City, New York. p. 7.

Division

Whether you realize it or not, the standard way of performing long division requires you to guess along the way until you find something that works. This new system is not only easier, it discards the guessing part. The rules are somewhat complicated, but the process is just as easy as adding, subtracting and multiplying. That is because, except for dividing small numbers, the rest of the process involves adding, subtracting and multiplying. I have tried to place the workspaces in an orderly fashion to guide you through the process. As before, the workspaces will eventually be transferred to your mind and disappear from view as "steps" in the process, leaving only the problem and the answer on display.

Let's look at a typical long division problem and a few variations to see how the rules will work.

884 / 34 = yy R zz. We will fill in the slots marked "y" with the answer and "z" with the remainder. The remainder is what is left after we cannot evenly divide any further. For now, the remainder will be a number that does not appear on your calculator because remainders are not included in the decimal system of a calculator. We will learn how to do that later.

Example 1

8 8 4 / **3** 4 =

1. Divide by the fewest numbers. In this case, the first digit of the dividend (884) can be divided by the first digit of the divisor (3). 8 / 3 = 6 with 2 as the remainder. Write it like this:

8 / 3 = 2 R 2

The result (2) becomes the first digit of the answer.

8 8 4 / 3 4 = **2**

2. Multiply the first digit of the answer (2) by what remains of the divisor, (4).

8 / 3 = **2** R 2
x 4
8

3. Combine the remainder (2) with the next number from the dividend (8). Subtract the product of step 2 (8) from this new number.

8 **8** 4 / 3 4 =

8 / 3 = 2 **R 2** **2 8**
x 4 **- 8**
8 **2 0**

Using this new dividend (20), repeat from step one (1) until we have reached the limit of the answer (in this case, two digits).

1. Divide 20 by 3. The result becomes the second digit of the answer.

8 8 4 / 3 4 = 2 **6** R 0

8 / 3 =		2	R	2		2	8
	x	4			-		8
		8				2	0

2 0 / 3 = 6 R 2

2. Multiply second digit of the answer (6) by the ones digit of the divisor (4).

2 0 / 3 =			**6**	R	2
	x		**4**		
		2	**4**		

3. Combine the remainder (2) with the next number from the dividend (4). Subtract the product of step 2 (24) from this new number.

8 8 **4** / 3 4 = 2 6

2 0 / 3 =			6	R	**2**		**2**	**4**
	x		4			**-**	**2**	**4**
		2	4					**0**

Because there are no more digits allowed for the answer, this subtraction process gives us the remainder from the problem, a zero (0).

8 8 4 / 3 4 = 2 6 **R 0**

These steps may seem complicated at first, but by working with small numbers and breaking down the problem into steps you can learn to do in your head, the process becomes easy and fast.

Long division may seem to be a long process when you see that it took two pages to demonstrate. We have presented the same information several times. When we show the problem as it would appear on your paper, it will not be as complex. In addition, remember that whatever you can do in your head will disappear from the workspace on paper as you gain accuracy and speed with practice. The steps shown here will train you to think so you can eventually perform most or all of the workspace in your head.

Problem, with answer:

8 8 4 / 3 4 = 2 6 R 0

Workspace:

0 8	/	3	=		2	R	2		2 8
				x	4			-	8
					8				2 0

---

2 0	/	3	=		6	R	2		2 4
				x	4			-	2 4
					2 4				0

Now let's look at some minor variations of this same problem. They will help you to understand how the process works a little better. Variations will also expose the only exception to the rules we have described. In the following examples, only those portions that differ from example 1 will be highlighted in red.

Variation A

Problem, with answer:

8 8 **5** / 3 4 = 2 6 R **1**

Workspace:

0 8	/	3	=		2	R	2		2 8
				x	4			-	8
					8				2 0

---

2 0	/	3	=		6	R	2		2 **5**
				x	4			-	2 4
					2 4				**1**

Variation B

Problem, with answer:
:

8 **9** **0** / 3 4 = 2 **7**

-1

2 **6** R **6**

Workspace:

8 / 3 = 2 R 2

x 4 = 8

2 **9** - 8 = 2 **1**

---

2 **1** / 3 = **7** R **0**

x 4 = **2** **8**

0 **0** - **2** **8**

3 **4** - **2** **8** = **6**

When we have a product that is too large to subtract, we must borrow one from our answer—in this case, from 7—and subtract our product from the entire divisor (34). 34 – 28 = 6, our remainder. Our adjusted answer becomes 26 instead of 27.

Example 2

Now that we have learned all the rules and the one exception, it is time to try a few more problems, to see how you do. This example would be a hard problem to perform the standard way. See how simple it becomes now!

Problem, with answer:

1 3 2 9 / 1 3 = 1 0 2 R 3

Workspace:

1	/	1	=	**1**	R	**0**	**0 3**
			x	**3**		-	**3**
				3			**0**

0	/	1	=	**0**	R	**0**	**0 2**
			x	**3**		-	**0**
				0			**2**

2	/	1	=	**2**	R	**0**	**0 9**
			x	**3**		-	**6**
				6			**3**

Compare this to the way you were taught in school:

```
            1  0  2    R   3
1  3  )  1  3  2  9
         1  3
            0  2
               0
               2  9
               2  6
                  3
```

There are similarities in the two processes that you may recognize. Our goal, however, is to develop a process that permits us to perform the problem in our head, using very easy steps. Once a child is successful with easy problems, he develops a sense of accomplishment and self-respect. When he learns that difficult problems can be handled by breaking them down into simpler steps, he is willing to accept greater challenges. Nothing is impossible for him because everything is built upon simple parts, repeated over and over. It becomes fun to do math. Are you having fun yet?

Example 3

Problem, with answer:

8 3 8 4 / 3 2 = 2 6 2 R 0

Workspace:

8	/	3	=	2	R	2		2 *3
			x	2			-	4
				4				1 9

---

1 9	/	3	=	6	R	1		1 8
			x	2			-	1 2
				1 2				6

6	/	3	=	2	R	0		0 4
			x	2			-	4
				4				0 0

Example 4

Problem, with answer:

4 7 9 5 3 5 / 6 3 = 7 6 1 1 R 4 2

Workspace:

```
4 7   /   6   =        7    R    5    5  9
                  x    3           -  2  1
                     2 1              3  8
```

```
3 8   /   6   =        6    R    2    2 *5
                  x    3           -  1  8
                     1 8                 7
```

```
  7   /   6   =        1    R    1    1  3
                  x    3           -     3
                       3              1  0
```

```
1 0   /   6   =        1    R    4    4  5
                  x    3           -     3
                       3              4  2
```

They just don't seem to get any harder, do they? We did find an exception—but only one. We have removed all of the guesswork and the large numbers out of the process of solving a long division problem. There are some added complexities ahead when we have divisors with more digits, but the process itself will follow these same rules. Become familiar with these rules and memorize your workspace as soon as possible.

When you practice doing problems in your head, always do them again on paper, to see how well you are remembering details. Don't worry if you forget some steps at first. This is part of the learning curve. By checking your work on paper, you will find where your mistakes happen most often and you can focus your study on those areas to correct them before they become bad habits.

Example 5

Problem, with answer:

$$\begin{array}{ccccccc} 1\ 9\ 0\ 4 & / & 3\ 4 & = & \begin{array}{r} 6\ 6 \\ -1 \\ \hline 5\ 6 \end{array} & R & 0 \end{array}$$

Workspace:

$$\begin{array}{ccccccccc} 1\ 9 & / & 3 & = & \begin{array}{r} 6 \\ \times\ \ 4 \\ \hline 2\ 4 \end{array} & R & 1 & \begin{array}{r} 1\ 0 \\ +\ 3\ 4 \\ \hline 4\ 4 \end{array} & \begin{array}{r} 4\ 4 \\ -\ 2\ 4 \\ \hline 2\ 0 \end{array} \end{array}$$

$$\begin{array}{cccccccc} 2\ 0 & / & 3 & = & \begin{array}{r} 6 \\ \times\ \ 4 \\ \hline 2\ 4 \end{array} & R & 2 & \begin{array}{r} 2\ 4 \\ -\ 2\ 4 \\ \hline 0 \end{array} \end{array}$$

The next examples have partial answers that total 10 or 11. They are easy to handle, however, because they do not cross to the next column as we did in addition, subtraction and multiplication. When you encounter a dividend of 10 or 11, put it into the single column where you will be working next. In every case, you will be subtracting enough times to reach 9 or below, a single digit. So these double digit figures are only temporary and they are never carried across to a neighboring column.

Example 6

Problem, with answer:

```
9 3 2 7 4    /    4 7    =    2  11   9
                             -1  -2  -1
                             ----------------------
                              1   9   8   4   R   2 6
```

Workspace:

```
 9    /    4    =       2    R    1      1 3       6 0
                   x    7              + 4 7     - 1 4
                   ------              -----     -----
                      1 4                6 0       4 6
```

```
4 6    /    4    =    1 1    R    2    2 2       6  9       1 *1 *6
                   x    7            + 4 7  +    4 `7   -      7  7
                   ------            -----    ---------   ----------
                      7 7              6 9    1  1  6          3  9
```

```
3 9    /    4    =      9    R    3     3  7       8 4
                   x    7             + 4 `7     - 6 3
                   ------             ------     -----
                      6 3               8  4       2 1
```

```
2 1    /    4    =      5    R    1     1  4       6 *1
                   x    7             + 4 `7     - 3  5
                                      ------     ------
                      3 5               6  1       2  6
```

Three of the four sections of our workspace require us to reduce the quotient. Section two requires us to reduce 11 twice, down to 9, adding the divisor (47) until we have a sum larger than the 77 we need to subtract.

What changes when we divide by more than two digits? As you might expect, the process becomes more complex. The good news is, there are no new rules. You only need to apply rules that you learned earlier in addition, subtraction and multiplication.

Example 7

Problem, with answer:

4 6 5 2 3 / 2 3 1 = 2 0 1 R 9 2

Workspace:

```
4    /    2    =    2    R    0       0 6
                                    -   6
   3 1 x 0 2  =    0 6              ------
                 +   0                  0
                 -----
                   0 6
```

```
0    /    2    =    0    R    0       0 5
                                    -   2
   3 1 x 2 0  =      0              ------
                 + 0 2                  3
                 -----
                   0 2
```

```
3    /    2    =    1    R    1                  1 2 3
                                               -   3 1
3 1 x 0 1        3       0       3 0           -------
3 1 x 1 0      + 0     + 1     +   1               9 2
               ---     ---     -----
                 3       1       3 1
```

You still divide by one digit of the divisor and multiply by the others—in this case, 31. And you don’t bring in new digits all at once, only what you need. Remember to reserve space in your answer for the number of digits you have in your entire divisor.

We only get one digit of the answer at a time. Use step two of your multiplication rules, (outside to inside and add) but multiply only the part of the answer you know. In section one, we know the first digit of our answer is 2. Multiply (rule two only!) 02 x 31. Section two gives us 20 x 31. Section three gives the final digit, 01 x 31.

In section three, the first multiplication gives us the tens digit of our answer (3). Put a zero in the ones digit. Use rule three and finish with the left side. Add (30 + 1) for a working total. Subtract 31 from the divisor to find the remainder.

Example 8

Problem, with answer:

```
3  4  4  8  5  8     /     5  3  8     =     6   5  1
                                                -1
                                             ____________________
                                             6   4  1     R     0
```

Workspace:

```
3  4      /     5     =       6     R       4        4  4
                                                   - 1  8
                                                     ____
  3  8  x  0  6             1  8                     2  6
                         +     0
                          _____
                            1  8
________________________________________________________________

2  6      /     5     =       5     R       1        1   8        7  *1
                                                   + 5  `3      -  6   3
                                                     ______     _________
  3  8  x  6  5             1  5                     7   1             8
                         +  4  8
                          _____
                            6  3
________________________________________________________________

   8      /     5     =       1     R       3        3  5  8
                                                   - 3  5  8
                                                     _______
  3  8  x  4  1       3  8  x  1  0                        0

        3                 0              3  5  0
  +  3  2              +  8           +     0  8
     ____                __             ________
     3  5                 8              3  5  8
________________________________________________________________
```

To keep the workspace orderly when you multiply twice in a section, put the additions below their corresponding multiplications. When there is only one multiplication in a section, you save space by putting the addition to the right, as shown above.

The next pages feature more examples, some with four-digit divisors. Again, these would be very difficult with any other math, but we make it so easy for you, your workspace could soon disappear.

Example 9

Problem, with answer:

```
6  3  8  7  3  9      /     4  8  2  6     =    1  3   3
                                                      -1
                                                ---------------------------
                                                1  3   2     R     1  7  0  7
```

Workspace:

```
 6     /     4     =        1     R       2     2  *3
                                              -     8
                                                 -------
 8  2  6  x  0  0  1       0  8                  1  5
                        +     0
                        +     0
                          -----
                           0  8
```

```
1  5     /     4     =        3     R       3     3  8
                                                -  2  6
                                                  -----
  8  2  6  x  0  1  3       2  4                  1  2
                         +     2
                         +     0
                           -----
                            2  6
```

```
1  2     /     4     =      3     R       0     0   7  3   9        5   5  6  5
                                              + 4  `8  2  `6      - 3   8  5  8
                                                -------------       -----------
                                                5   5  6   5        1   7  0  7

                          2   4                                           0
                       +      6                                    +      6
8  2  6  x  1  3  3    +     `6       8  2  6  x  3  3  0          +  1  `8
                         ------                                      ------
                          3   6                                       2   4

                              0             3  6  0  0
                       +      0         +      2  4  0
8  2  6  x  3  0  0    +  1   8         +         1  8
                         ------           ------------
                          1   8             3  8  5  8
```

Example 10

Problem, with answer:

5 6 2 8 7 6 5 1 / 8 3 7 1 9 =

7 8 3
-1 -1 -1
6 7 2 R 2 8 4 8 3

Workspace:

5 6 / 8 = 7 R 0

0 2
+ 8 3
8 5

8 5
- 2 1
6 4

3 7 1 9 x 0 0 0 7 = 2 1

---

6 4 / 8 = 8 R 0

0 8
+ 8 `3
9 1

9 *1
- 6 6
2 5

3 7 1 9 x 0 0 6 8

2 4
+ 4 2
6 6

---

2 5 / 8 = 3 R 1

1 7 6 5 1
+ 8 `3 `7 1 9
1 *0 *1 *3 *7 *0
- 7 2 8 8 7
2 8 4 8 3

3 7 1 9 x 0 6 7 3

3 7 1 9 x 6 7 3 0

3 7 1 9 x 7 3 0 0

3 7 1 9 x 3 0 0 0

9
+ 4 `9
+ `6
+ 0
6 4

0
+ 2 1
+ 7
+ 5 4
8 2

0
+ 0
+ 3
+ 6 3
6 6

0
+ 0
+ 0
+ 2 7
2 7

6 4 0 0 0
+ `8 2 0 0
+ 6 6 0
+ 2 7
7 2 8 8 7

NOTE: First answer (7) multiplied in section one. In section 2, we know it was reduced, so we now multiply 68. For the third section, we use 673, since we had reduced the 8.

Division is the last process in our study. We have presented simple rules for addition, subtraction, multiplication and division. At this point, if you have not done so, you should create problems of your own and solve them. Drill is what will make you successful in basic math, regardless of which rules you learn. The rules you learned here, however, will make it possible to do much more of each problem in your head instead of on paper. This will eventually save much time, because when you can perform portions in your mind, it will be much faster that writing it on paper. Thus very large problems can be done with a minimum amount of writing, and the final solution happens very fast.

How will you know when you have "arrived"? What will be your clue that you are a math whiz?

If you can solve one of these problems nearly as fast in your head as someone who is entering the numbers into a calculator for the answer, you have "arrived." People who cannot do what you can do will be amazed and call you a "human calculator." You and I will know the secret to your success. They will know, too, if we tell them. But until then, they will think of you as a mental giant.

I like that feeling. Would you like to have that, too?

Soon I hope to add a computer application that will provide random drill exercises and track how much time it takes you to solve a problem. Return now and then to www.xtreme-ed.com to find out when that application will be released.

If you are already familiar with math, and you're just trying to improve your ability, I'd like to propose a challenge. If you'd like, keep track of your progress and send me a report.

Before you begin serious study of this process, create some sample problems. Make them difficult enough to be interesting. Do at least 10 problems in addition, another 10 in subtraction, etc. Create more if you wish, but you should do all the addition problems in one sitting. Keep track of your start and finish times. Take a break between addition, subtraction, multiplication and division, even if you do all the problems in one day.

After you are confident that you have improved both speed and accuracy, do these same problems again using these rules, again keeping track of the start and end times. Then let me know 1) how many hours of study you put in to arrive at this point, 2) the amount of improvement in speed and 3) how you feel about your work thus far. In fact, I would like to know what your profession is, and how you use arithmetic in your daily life, if you don't mind telling me.

Now drill, drill, drill. After you know the rules, that's what it will take to increase your speed and improve your mental prowess.